VEGETATION INFORMATION GUIDELINES

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Introduction:

Please read these guidelines carefully and completely before initiating any vegetation studies.

These guidelines are intended to provide a suggested format for the submittal of vegetation information to be included in the mining and reclamation plans for coal mining operations. The purpose of submitting such information is as follows:

1. To approximate and describe the vegetative resources prior to mining;

2. To identify and describe important wildlife habitat areas;

3. To identify and provide protection for any listed or proposed threatened and/or endangered plant species;

4. To aid in the prediction of revegetation potential for the site; and

5. To identify the standards or methodology by which the success of revegetation will be measured for the purpose of bond release.

Should problems or questions arise concerning these guidelines, contact the Division of Oil, Gas and Mining.

DEFINITIONS:

Adjacent Areas: Areas outside the permit area that are within 1/2 mile of areas that will be affected by mining operations.

Baseline Data: Data collected to describe the "original" (pre-disturbed) condition of a vegetation type or ecological site.

Cover by Species: The percent of ground covered by a species or life form (cover by species may and often does add up to more than 100% and is used to establish plant diversity). Non-native species may be included in the count, however, undesirable species should be separated from the desired species to determine living cover of desirable vegetation, used to achieve the post-mining land use.

Density: The number of plants per unit of area.

Diversity: The measure of variations of species in a given area. This can be measured using the Shannon's Diversity Index and should not include non-native or noxious weed species.

Ecological Site: A distinctive kind of land with specific physical characteristics which differs from other kinds of land in its ability to produce distinctive kinds and amounts of vegetation and in its response to management. Apparently synonymous with ecological type used by USDA Forest Service. Syn. Rangeland ecological site.

Ecological Site Description: Description of soils, uses and potential of a kind of land with specific physical characteristics to produce distinctive kinds and amounts of vegetation.

Ground Cover: The percent of ground covered by vegetation, regardless of species. Ground cover cannot exceed 100% when added to the percent of aerial projection of rock, litter, and bare ground.

Normal Precipitation Year: A year where the effective precipitation is 90% of the 10-year average and within 90% of the 10-year monthly average for the month prior to sampling. Effective precipitation is that which falls from October 1 of the previous year to the end of the month prior to sampling.

Productivity (Production): The average yield of food, fiber, forage and/or wood products per unit of area per year.

Random Sample: A sample taken such that any point in the sample area has an equal chance of being sampled at any time during the sampling sequence.

Reference Area: An area that is similar to the community to be disturbed with respect to vegetation (cover, density, composition), soils, aspect, climate, and elevation that will be maintained and used as the standard for comparisons with the reclaimed "disturbed' area.

Species Composition: The species found within a given area.

Vegetation Type: A plant community that is distinguished by its visually dominant species and should be described by two or more dominant species.

Woody Plants: Those plants which are classified as sub-shrubs, shrubs, half-trees or trees.

SUGGESTED STEPS IN PREPARING PREMINING VEGETATION INFORMATION:

1. Map the existing vegetation types (or ecological sites) found within the permit area and adjacent areas {scale of 1:6,000 (1"=500') or larger}. Show the locations and boundaries of current disturbed areas as well as any areas proposed to be disturbed, the locations of any listed or proposed threatened or endangered plant species, the locations of sample points and the locations and boundaries of any proposed vegetation reference areas. Vegetation type boundaries should overlay the disturbance areas. Map all potentially disturbed areas on contour maps of a scale approved by the Division {1:2,400 (1'=200') scale or larger}. Mark these maps so that referral maybe made back to the permit area map. Aerial photographs of sufficient scale would be acceptable for mapping requirements. Map requirements may be altered on a case-by-case basis by contacting the Division in advance.

2. Determine and list the acreage of each vegetation type (or ecological site) to be disturbed or that has been disturbed. Note the total acreage of surface disturbance (existing and proposed) within the permit area.

3. In a narrative, describe each vegetation type (or ecological site) by visually dominant species, and describe the condition and relative stage of maturity of the vegetation type. Note any past perturbations in the area such as fire, chaining, reseeding, previous mining, cultivation, etc. Discuss any present use by wildlife or livestock and correlate each vegetation type with wildlife habitat types or wildlife use areas. Provide a statement of productivity (a letter of assessment from the Natural Resources Conservation Service would suffice). For forest types, provide an estimate of wood volume.

4. Sample each vegetation type (or ecological site) which exists within the proposed disturbed areas or was assumed to have existed within existing disturbed areas according to the methodology selected for determining revegetation success (reference area method, ecological site method or baseline data method). Approved sampling techniques must be used (see Appendix A). Sampling should be done during the height of the growing season (usually late June to late August). It is highly recommended that a site visit be arranged between the Division and the persons responsible for data collection prior to vegetation sampling.

5. List all native and non-native species present within each vegetation type (or ecological site) or any proposed reference area by common and botanical name. List the species by plant groupings, i.e., trees, shrubs, forbs, grasses, etc. Identify noxious species encountered.

6. Identify any listed or proposed threatened, endangered, or sensitive plant species that occur on the permit or adjacent areas. Make a negative declaration if these are not found. A current list of listed or proposed threatened or endangered species can be obtained from the U.S Fish and Wildlife Service.

7. Correlate vegetation reference areas, ecological sites or vegetation types with revegetation plans and the proposed postmining land use for all affected areas. Reference areas must be approved by the Division.

8. As per R614-301-356.231, if fish and/or wildlife habitat, recreation, shelterbelts or forest products are to be a primary or secondary use, the Division will provide, in technical memoranda, evidence of consultation and acceptance of proposed woody plant stocking densities with the Utah Division of Wildlife Resources and other appropriate land and wildlife management agencies.

9. All technical data submitted in the application shall be accompanied by:

    a. The names of persons or organizations which collected and analyzed such data;

    b. The dates of data collection and analysis;

    c. Descriptions of methodology used to collect and analyze data (including means, standard deviations, formulae used, etc.); and

    d. The name, address and position of officials of each private or academic agency consulted by the applicant in preparation of the information.

METHODS:

1. Reference Areas:

The use of vegetation reference areas for establishing revegetation success standards are applicable to all mining situations. For mines with new disturbance (either new mines or existing mines with proposed new disturbance), reference areas are selected and compared with the vegetation existing on the area to be disturbed. For areas of existing disturbance, the reference areas are selected on the basis of the vegetation that most likely existed prior to disturbance. Generally, a reference area is needed for each major vegetation type that has been or will be disturbed. All reference areas must be approved by the Division prior to using them for revegetation success standards (this may be done prior to permit approval). Reference areas do not need to be established for types where less than 1 acre will be disturbed or where the community type will be greatly altered by an approved postmining land use. However, a revegetation success standard must still be established for these areas. Reference areas should be at least 1 acre in size unless otherwise approved by the Division in advance.

For Division approval, the following information must be provided for each vegetation type that will be disturbed and the corresponding proposed reference area (or any proposed reference areas for existing disturbed areas):

a. Randomly sample for ground cover, cover by species, woody plant density and productivity. Productivity measurements need not include noxious weeds (a list of noxious weeds may be obtained from the County Weed Supervisor, U.S.U. Extension Service or the District Agriculture Inspector).

b. Assess the current ecological site condition of the affected areas as well as all proposed reference areas (ecological site condition should be re-assessed every 5 years, during the field season prior to permit renewal). Ecological site condition should be determined according to Natural Resource Conservation Service (NRCS) guidelines. Reference areas must be categorized as a moderate to slight, or slight to none regarding its departure from expected range conditions according to the NRCS guidelines. For reference areas not meeting this criteria, describe management practices (i.e., fencing) that will be employed to improve site condition. Ecological site condition should be determined according to Soil Conservation Service guidelines.

c. Demonstrate sample adequacy for ground cover, density (woody plants), and productivity (see Appendix A).

d. Demonstrate by table (see attached summary table example), or other simplified format, the similarity between proposed disturbed sites and the corresponding proposed reference area according to the following parameters:

   i. Species composition (by a similarity index, see Appendix B), similarity should be 70% unless otherwise approved by the Division.

   ii. Ground cover and woody plant density (by a t-test).

   iii. Productivity, soils, slope, aspect, and land use.

e. One reference area may represent more than one disturbance site if the reference area meets the requirements for each site.

f. Mark off the proposed reference areas in the field with permanent, readily visible markers (i.e., t-posts) so that they can be easily located.

g. Upon request, submit to the Division copies of the data sheets from sampling of areas to be disturbed and potential reference areas.

After receiving approval from the Division for the reference site, the range condition of the reference site must be re-assessed every 5 years during the field season prior to permit renewal.

2. Ecological Sites:

In order to use ecological sites as an alternative to vegetation reference areas for revegetation success standards, the following criteria **must** be met:

a. Ecological sites must be described in accordance with the Natural Resource Conservation Service's National Range and Pasture Handbook, as amended.

b. Ecological sites to be sampled must have a moderate to slight, or slight to no departure from expected ecological site conditions according to the NRCS guidelines and the site must be representative of the plant communities that will exist after reclamation according to the approved post mining land use.

c. Sampling must be done during a normal precipitation year.

d. The ecological site area to be sampled must be at least one acre in size.

For each ecological site that will be or has been disturbed:

a. Randomly sample for ground cover, cover by species, woody plant density and productivity. Productivity measurements need not include noxious weeds.

b. Assess the current ecological site condition (if the condition is not categorized as a moderate to slight, or slight to no departure from expected range conditions, the ecological site method cannot be used). Ecological site condition is determined according to Natural Resources Conservation Service guidelines.

c. Demonstrate sample adequacy for ground cover, density (woody plants) and productivity (see Appendix A).

Since the results of this sampling will be considered the values for the success standard for revegetation success, a legible copy of all data sheets must be submitted to the Division.

3. Baseline Data

The baseline data method can only be used for areas of proposed new disturbance. In order to use baseline data as an alternative to vegetation reference areas for revegetation success standards, the following criteria must be met:

a. The data must be collected from the proposed disturbed area(s).

b. Vegetation types to be sampled must be categorized as a moderate to slight, or slight to no departure from expected ecological site conditions according to the NRCS guidelines and the area must be representative of the plant communities that will exist after reclamation according to the approved post mining land use.

c. Sampling must be done during a normal precipitation year.

For each vegetation type that will be or has been disturbed:

a. Randomly sample for ground cover, cover by species, woody plant density and productivity. Non-native and noxious weed species, if present, should be included and separated in the report as to

clearly indicate the proportion of non-native or noxious weed species at the site. Undesirable non-native and noxious weed species cannot be used in success standards.

b. Assess the current range condition ecological site condition (if the condition is not categorized as a moderate to slight or slight to no departure from expected ecological site conditions, the baseline data method cannot be used). Ecological site condition should be determined according to the NRCS guidelines.

c. Demonstrate sample adequacy for ground cover, density (woody plants) and productivity (see Appendix A).

Since the results of this sampling will be considered the values for the success standard for revegetation success, a legible copy of all data sheets must be submitted to the Division.

SUMMARY OF MAP GUIDELINES:

A vegetation map of the entire permit area and adjacent areas on a scale of 1:6,000 (1'=500') or larger should be submitted if not otherwise exempted by the Division. A contour map {scale of 1:2,400 (1'=200') or larger} should be submitted for all areas of present or potential disturbance.

The permit area map should show the boundaries and/or locations of:

1. The permit area and give the legal description {i.e. township, range, and section(s)};

2. Any surface areas which are disturbed by mining or any areas proposed to be disturbed;

3. Any proposed vegetation reference areas;

4. Existing vegetation types or range sites;

5. Any listed or proposed threatened or endangered plant species; and

6. Sampling sites.

The disturbed area map(s) should:

1. Provide reference points back to the permit area map, including legal description;

2. Show the vegetation types or range sites which currently exist in areas of proposed disturbance, or which are assumed to have existed in current disturbed areas; and

3. Show the locations of sampling sites.

VEGETATION DATA SUMMARY

(- Company -)
(- Mine Name -)
(- Permit Number -)

VEGETATION TYPE:_____ Date Data Collected:_____

% Similarity between Reference and Affected Areas:_____ Index Used:_____

	REFERENCE AREA				CORRESPONDING AREA TO BE AFFECTED				
	$\bar{X}$	S	N	N_{min}	$\bar{X}$	S	N	N_{min}	't'
Ground Cover									
Density									
Productivity									
Soil Type									
% Slope									
Aspect									
Land Use									

$\bar{X}$ = Sample Mean
S = Standard Deviation
N = Sample Size

N_{min} = Minimum Sample Size (for statistical adequacy)
't' = Calculated t - value from t - test

VEGETATION INFORMATION GUIDELINES

APPENDIX A

(February 1992)

ACCEPTABLE SAMPLING METHODS FOR VEGETATION STUDIES

Pursuant to R645-301-356.110, the following sampling methods, as described below, have been selected and approved by the Utah Division of Oil, Gas and Mining for conducting vegetation studies for permitting purposes and for determining revegetation success of reclaimed areas. One should select the most appropriate sampling method for the community to be sampled. Sampling methods other than those described herein must be submitted through the Division to OSM for review as a state program amendment. Any sampling method used, whether identified in this guideline or approved as an alternative method, must be described in detail and approved as part of the permit application package or as an amendment thereto.

For sampling methods that require the use of quadrats, please note that quadrat size and shape are not fixed. However, common use is made of rectangular or square plots of m^2, $^1/_4\, m^2$, or 20 X 50cm in size or a $^1/_4\, m^2$ - m^2 circular plot.

I. Sample Adequacy

Regardless of sample size requirements determined from the formula below, the minimum sample size listed for each method must be achieved. All other sampling must meet the statistically adequate sample size as determined by the formula:

$$N_{min} = \frac{t^2 s^2}{(dx)^2}$$

where: t = the value from appropriate t-table*, (2-tail test for premine studies, 1-tail test for revegetation success studies),

s = the sample standard deviation,

d = the desired change in the mean,

x = the sample mean of the parameter in question.

* All parameters should be tested at the 90% confidence level with a 10% change in the mean (d= .1)

Reference: Cochran, W.G., 1977. Sampling techniques, 3rd ed. John Wiley and Sons, New York, N.Y. 428pp.

II. COVER

1. Ocular Estimation:

Estimate the percent of ground covered by vegetation (by species, total vegetation, litter, rock, etc.) to the nearest percent. Values should be reported by species (and could potentially exceed 100% due to overlap) and by total vegetation cover. Total vegetation cover, when added to vertical projection of exposed rock, litter and bare ground will equal 100%. Each quadrat is considered one sampling unit. Since ocular estimation is more subjective than exact measurement methodologies, ideally, sampling would be done by the same individual to promote consistency between monitoring years.

Quadrats should be randomly placed within the study area.

Minimum sample size = 10

Reference: Daubenmire, R., 1959. A Canopy-Cover Method of Vegetational Analysis. Northwest Science 33:43-63.

2. Cover Classes:

Cover classes may be used, provided they are at least as small (in range) as those listed below. Utilizing quadrats as discussed above, estimate the percent of ground covered by vegetation to the nearest class. Values should be reported by species and total vegetation cover. Each quadrat is considered one sampling unit. When analyzing the data, the mid-point of each class is used to calculate the mean and standard deviation.

Cover Class	Range	Mid-Point	Cover Class	Range	Mid-Point
1 =	0-1.0%	.5%	8 =	35.1-45.0%	40.0%
2 =	1.1-3.0%	2.0%	9 =	45.1-55.0%	50.0%
3 =	3.1-5.0%	4.0%	10 =	55.1-65.0%	60.0%
4 =	5.1-10.0%	7.5%	11 =	65.1-75.0%	70.0%
5 =	10.1-15.0%	12.5%	12 =	75.1-85.0%	80.0%
6 =	15.1-25.0%	20.0%	13 =	85.1-95.0%	90.0%
7 =	25.1-35.0%	30.0%	14 =	95.1-100%	97.5%

Minimum Sample Size = 20

Reference: Daubenmire, R., 1959. A Canopy-Cover Method of Vegetational Analysis. Northwest Science 33 :43-63.

3. Point Methods:

Vegetation cover is identified at a pre-determined 'point' and recorded as vegetation, litter, rock or bare ground. Points may be located systematically or randomly along a tape, using a pin frame or an ocular devise with cross hairs. Total vegetation cover is determined by the first interception or hit (i.e., vegetation, rock, litter, etc.) Cover by species is determined by subsequent hits of vegetation as the point (pin) is lowered through the vegetation. Transects of 50 point minimum are counted as one sample unit. The location and orientation of the transect within the study site should be randomly placed.

Minimum sample size = 15

Reference: Goodall, D.W., 1943. Point Quadrat Methods for the Analysis of Vegetation. The Treatment of Data for Tussock Grasses. Aust. J. Bot. 1 :457-461.

4. Line Interception:

Using the line intercept method, percent cover is obtained by summing the distances of the transect that are covered by vegetation, litter, rock, bare ground. Transects are commonly 10-100m long. Each transect is counted as one sampling unit. Transects should be randomly placed within the study area. (This method is best used in sparse, low vegetation.)

Minimum sample size = 15

Reference: Canfield, R.H., 1941. Application of the Line Interception Method in Sampling Range Vegetation. J. For. 39 :388- 394.

III. DENSITY (SHRUBS AND/OR TREES)

1. Point-Quarter Method:

Randomly locate sample points within the study area. At each point, two lines are made to divide the area into four quarters, with the point being the center. The distance from the point to the base of the nearest plant in each quarter is then measured and recorded. To determine the density, sum the 4 distances measured at each point and divide by 4. This mean distance is then squared to give the mean area per plant (this is done for each sampling point). Sum the mean area per plant of each point and divide by the number of points sampled. Divide 43,560 by this number to obtain plants per acre (formulas summarized below). This is the preferred method for semi-dense to dense stands.

Points may be randomly located in the stand or along randomly located transects.

Minimum sample size = 10

Density Formula

For each point:

$$A_j = \left(\frac{\sum Y_i}{4}\right)^2 \qquad D = 43{,}560 + \frac{\sum A_j}{n}$$

Where: Y_i = Distance from point to nearest plant in the i^{th} quarter.

A_j = mean area/plant at the j^{th} point.

n = sample size (number of points sampled).

D = plants/acre.

Reference: Cottam, G., and J.T. Curtis, 1956. The Use of Distance Measures in Phytosociological Sampling. Ecology 37(3):451-460.

2. Belt Transects or Plots:

Belt transects or plots are randomly placed in the study area and the number of plants that are rooted in each plot are counted, even if all of the plant canopy is not within the plot. Likewise, plants that overlap the plot but are not rooted within the plot are not counted. The size of the plot is not fixed; however, those sizes commonly used are: M2, 5ft-10ft x 100ft, .1 acre, or 1-5m x 50m. Each plot is counted as one sample unit. Select the plot size that is best suited to the community being sampled. This method is better adapted for low to semi-dense stands.

To obtain the number of plants/acres, multiply the number of plants counted in the plot by 43,560 and divide the product by the size of the plot (in square feet).

Minimum sample size = 15

Reference: Chambers, Jeanne C., Ray W. Brown, Methods for Vegetation Sampling and Analysis on Revegetated Mined Lands. Gen. Tech. Rep. INT-151. Ogden, Utah: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Exp. Station; 1983, 57p.

3. Exact Count:

For Extremely Small Stands (usually less than 1 acre) or Very Low Density Areas, an exact count may be preferred since the use of an exact count is not subject to statistical tests of sample adequacy.

IV. PRODUCTIVITY MEASUREMENTS

1. Exclosures:

The use of exclosures for productivity measurements is optional where domestic livestock will not be in the study area prior to sampling. If livestock are to be in the study area prior to sampling, then exclosures should be used.

When used, exclosures should be large enough to prevent animals from reaching through and grazing on the plot to be sampled. Exclosures should be randomly placed and anchored to the ground, before the growing season begins. The number of exclosures established should be based on previously collected production data and field experience. To reduce variability and sample sizes, community types should be separated as much as possible. Exclosures should be numbered in the order of the random numbers generated for their placement. Sampling should follow the number sequence until sample adequacy is met or all exclosures have been sampled.

2. Clipping:

Select the quadrat size that is best suited to the community being sampled. Randomly locate the quadrat and clip plants by life form (e.g., herbaceous or woody). For grasses and forbs, clip all standing biomass; for shrubs, clip only current year's growth. Oven dry samples and weigh to the nearest .1 gram. For sample adequacy, use the combined weight of each life form at each plot. Report productivity as pounds/acre or kilograms/hectare.

Minimum sample size = 10 quadrats

Reference: Boyer, William D.: 1059. Harvesting and Weighing Vegetation. Pages 11 through 16. In: Techniques and Methods of Measuring Understory Vegetation. USDA Forest Service Southern Forest Exp. Station and Southeastern Forest Exp. Station.

3. Double Sampling:

Select the quadrat size that is best suited to the community being sampled. 2-4 quadrats are clustered systematically around a central, randomly located quadrat. The amount of biomass in the clustered quadrats is estimated as a percent of the biomass of the center quadrat. The center quadrat is then clipped, dried and weighed. A weight is then calculated for the clustered plots based on the percent recorded. For testing purposes, the mean weight for the cluster is used with each cluster being counted as one sample unit. Report productivity as pounds/acre or kilograms/hectare.

Minimum sample size = 10

Reference: Boyer, William D., 1059. Harvesting and Weighing Vegetation. Pages 11 through 16. In: Techniques and Methods of Measuring Understory Vegetation. USDA Forest Service Southern Forest Exp. Station and Southeastern Forest Exp. Station.

4. Natural Resources Conservation Service Estimation:

For establishing reference areas, it is preferred that the Natural Resources Conservation Service be contacted to estimate productivity and evaluate range condition. Their signed statement will be sufficient for the premining inventory for production on the affected area and reference area.

VEGETATION INFORMATION GUIDELINES

APPENDIX B
(August 2024)

ACCEPTABLE SIMILARITY / DIVERSITY INDICIES

1. Jaccard's Community Coefficient:

$$SI = \frac{common\ species}{total\ species}\ x\ 100 \quad \text{or} \quad SI = \frac{c}{a+b-c}\ x\ 100$$

Where: SI = Similarity index;

a = Total number of species in community a;

b = Total number of species in community b; and

c = Number of species common to both communities.

Reference: Jaccard, P., 1912. The Distribution of the Flora of the Alpine Zone. New Phytologist 11 :37-50.

2. Ruzicka's Index of Quantitative Similarity:

{Quantitative data is required for this index (i.e., cover or productivity by species).}

$$SI = \frac{\sum min}{\sum max}\ x\ 100$$

Where: SI = Similarity index;

∑min = Sum of minimum values for any species in the two communities (zero is possible); and

∑max = Sum of maximum values for any species in the two communities.

Reference: Ruzicka, M., 1958. Anuendung Mathematisch - Statisticher Methoden in Der Geobotanik (Synthetische Bearbeitung von Aufnahmen). Biologia, Bratisl. 13 :647-661.

3. Sorensen's Similarity Index:

$$SI\ = \frac{2C}{A+B}\ x\ 100$$

Where: SI = Similarity index;

A = Total number of species in community A;

B = Total number of species in community B; and

C = Number of species common to both communities.

Reference: Sorensen, T., 1948. A Method of Establishing Groups of Equal Amplitude in Plant Sociology Based on Similarity of Species Content. Det Kong. Danske Vidensk. Selsk. Biol. Skr. (Copenhagen) 5:1-34.

4. Shannon's Diversity Index

(To be used to demonstrate diversity of the reclaimed area, *not* as a comparison to the reference area.)

$$H = -\sum_{i=1}^{S} p_i \ln p_i$$

Where: H = Shannon's Diversity Index
P_i = the proportion individuals belonging to i-th species

Reference: Chambers, Jeanne C., Ray W. Brown, Methods for Vegetation Sampling and Analysis on Revegetated Mined Lands. Gen. Tech. Rep. INT-151. Ogden, Utah: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Exp. Station; 1983, 57p.

5. One-sample, one-sided t-test

This is appropriate for comparing a normally – distributed parameter to a technical standard.

$$t^* = \frac{\bar{x} - 0.9(technical\ standard)}{s/\sqrt{n}}$$

Where: t^* = calculated t-statistic.

$\bar{x}$ = sample mean.

s = standard deviation of the sample.

n = sample size

The α – level of the test is 0.10 by regulation and the decision rules for testing the reverse null hypothesis are:

If $t^* < t(1-\alpha\,; n-1)$, conclude failure to meet performance standard.

If $t^* \geq t(1-\alpha\,;n-1)$, conclude the performance standard was met.

Reference: Neter, J., Wasserman, W., and Kutner, M. H. 1985. Applied Linear Statistical Models, 2nd ed. Irwin Press, Homewood, IL 60430. 1127 pp.

6. One-sided t-test for two independent samples

This test is appropriate for comparing samples from two independent normally – distributed populations.

$$t^* = \frac{\overline{x_1} - 0.9\overline{x_2}}{\sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{0.81}{n_2}\right)}}$$

Where: t^* = calculated t-statistic.

$\overline{x_1}$ = reclamation sample mean.

$\overline{x_2}$ = reference area sample mean.

SS_1 = reclamation sum of squared deviations from the mean $\{_\phi(x_{1j} - 0_1)^2\}$.

SS_2 = reference area sum of squared deviations from the mean $\{\sum(x_{2j} - 0_2)^2\}$.

n_1 = reclamation sample size.

n_2 = reference area sample size.

The α – level of the test is 0.10 by regulation and the decision rules for testing the reverse null hypothesis are:

If $t^* < t(1-\alpha\,;n_1 + n_2 - 2)$, conclude failure to meet performance standard.

If $t^* \geq t(1-\alpha\,;n_1 + n_2 - 2)$, conclude the performance standard was met.

Reference: Neter, J., Wasserman, W., and Kutner, M. H. 1985. Applied Linear Statistical Models, 2nd ed. Irwin Press, Homewood, IL 60430. 1127 pp.

7. Mann – Whitney test for two independent samples

The Mann – Whitney test is appropriate for testing whether two populations have the same median values for a parameter. The populations need not follow a normal distribution, although it is assumed that the two populations have the same distribution; that is, the population variances are assumed to be equal. The Mann – Whitney test is especially apt in cases where two long – tailed sample distributions are being compared, because comparisons of observation ranks, rather than actual values, are made.

The first consideration in the bond release scenario is how to incorporate the 90% performance standard into the test. We wish to detect a shift in hypothesized population median, rather than a multiplicative effect. A transformation of both reclaimed and reference data must be made prior to assigning ranks. Since ranks are invariant to logarithmic transformations, the log transformation is an appropriate choice.

$$X'_{reference} = log\left(X_{reference} + 1\right) + log(0.9)$$

Remember the log(xy) = log(x) + log(y). The 1 is added to the observation values in case some observations are equal to zero, since log(0) is undefined.

$$X_{reclamation} = log(X_{reclamation} + 1)$$

We then combine all of the log – transformed values from both samples and rank them from the smallest (which is given a rank of 1) to the largest. Tied observations are assigned the average of the ranks they would have received if there were no ties. We then sum the ranks of the transformed observations from the reference area population ($S_{reference}$).

$$T = \left(S_{reference}\right) - \left(\frac{n_1(n_1 + 1)}{2}\right)$$

The probability of observing a value of z is shown in Table B-3, and we can conclude whether the performance standard was met.

$$z = \frac{T - {n_1 n_2}/{2}}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}}$$

Where:

$S_{reference}$ = reference area population.

T = test statistic.

n_1 = number of observations in the reference area sample.

n_1 = number of observations in the reclamation sample.

$w_{0.10}$ = critical value of T observed in Table B-2 given n_1 and n_2.

The decision rules, with α set at 0.10 are:

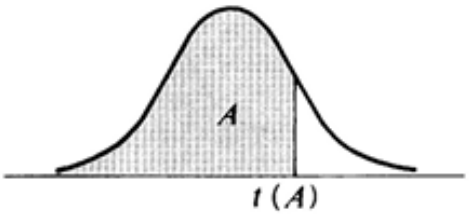
If $T > w_{0.10}$, conclude failure to meet performance standard.

If $T \leq w_{0.10}$, conclude the performance standard was met.

Reference: Daniel, W.W. 1990. Applied Nonparametric Statistics, 2nd ed. PWS-KENT PublishingCo., Boston, MA. 635 pp.

Table B-1: Percentiles of the t distribution

Entry is $t(A;\nu)$ where $P\{t(\nu) \leq t(A;\nu)\} = A$



	A						
ν	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

*Table B-1 continued on page 19.

	A						
ν	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

Reference: Neter, J., Wasserman, W., and Kutner, M. H. 1985. Applied Linear Statistical Models, 2nd ed. Irwin Press, Homewood, IL 60430. 518 pp.

Table B-2: Values of $w_{0.10}$ for Mann – Whitney test statistic

n_1	n_2=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2		0	1	1	2	2	2	3	3	4	4	5	5	5	6	6	7	7	8	8
3		1	2	2	3	4	5	6	6	7	8	9	10	11	11	12	13	14	15	16
4		1	2	4	5	6	7	8	10	11	12	13	14	16	17	18	19	21	22	23
5		2	3	5	6	8	9	11	13	14	16	18	19	21	23	24	26	28	29	31
6		2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	35	37	39
7		2	5	7	9	12	14	17	19	22	24	27	29	32	34	37	39	42	44	47
8		3	6	8	11	14	17	20	23	25	28	31	34	37	40	43	46	49	52	55
9		3	6	10	13	16	19	23	26	29	32	36	39	42	46	49	53	56	59	63
10		4	7	11	14	18	22	25	29	33	37	40	44	48	52	55	59	63	67	71
11		4	8	12	16	20	24	28	32	37	41	45	49	53	58	62	66	70	74	79
12		5	9	13	18	22	27	31	36	40	45	50	54	59	64	68	73	78	82	87
13		5	10	14	19	24	29	34	39	44	49	54	59	64	69	75	80	85	90	95
14		5	11	16	21	26	32	37	42	48	53	59	64	70	75	81	86	92	98	103
15		6	11	17	23	28	34	40	46	52	58	64	69	75	81	87	93	99	105	111
16		6	12	18	24	30	37	43	49	55	62	68	75	81	87	94	100	107	113	120
17		7	13	19	26	32	39	46	53	59	66	73	80	86	93	100	107	114	121	128
18		7	14	21	28	35	42	49	56	63	70	78	85	92	99	107	114	121	129	136
19		8	15	22	29	37	44	52	59	67	74	82	90	98	105	113	121	129	136	144
20		8	16	23	31	39	47	55	63	71	79	87	95	103	111	120	128	136	144	152

Adapted from Daniel, W.W. 1990. Applied Nonparametric Statistics, 2nd ed.

Table B-3: Standard one-tailed normal curve area

Table entries give the area under the normal curve from 0 to *z*. Subtract the table entry from 0.5 to obtain the tail area of the curve, which is the probability of randomly observing a value of *z* which is equal to, or more extreme than, the calculated *z* value. If calculated values have negative signs, disregard the sign when using this table. For example, the table entry for $z = -1.96$ is 0.4750, and the probability of randomly observing that *z* value is 0.0250.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2133	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Adapted from Snedecor, G.W., and Cochran, W.G. 1980. Statistical Methods, 7th ed.